

Complex Numbers

33. Let $z = 1 + i$.

- (a) Express the complex number z^{50} in polar form, and calculate its real and imaginary components.
- (b) Express the complex number z^{-1} in polar form, and calculate its real and imaginary components.
- (c) The complex number $z = 1 + i$ can be represented by the point $T(1, 1)$ in the complex plane. Explain the geometric meaning of the product $z \cdot \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$ with respect to the point T . What is the geometric meaning of the product $z \cdot \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$ with respect to the point T ?

34. Solve the equation in the set of complex numbers:

$$z^2 = 2i.$$

Write all solutions in the form $a + ib$, where a and b are real numbers, and also in polar form. Additionally, plot all solutions in the complex plane with rectangular coordinates.

35. Find the solutions to the equation in the set of complex numbers \mathbb{C} :

$$2z^2 + 1 = 2i \operatorname{Im}(z).$$

Write the solutions in polar form.

36. Solve the equation in the set of complex numbers:

$$z^3 = -4.$$

Write all solutions in the form $a + ib$, where a and b are real numbers, and also in polar form. Additionally, plot all solutions in the complex plane using rectangular coordinates.

37. Find all complex numbers z that satisfy the equation $z^8 = 1$. Write all solutions in polar form $r(\cos \varphi + i \sin \varphi)$, where $r \in \mathbb{R}$, $\varphi \in [0, 2\pi)$, and in rectangular form $x + iy$, where $x, y \in \mathbb{R}$. Additionally, plot all solutions in the complex plane using rectangular coordinates.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.